A New Analytical Expression for the Relationship Between the Charpy Impact Energy and Notch Tip Position for Functionally Graded Steels

H. Samareh Salavati Pour^{1,2)†}, F. Berto³⁾ and Y. Alizadeh¹⁾

1) Department of Mechanical Engineering, Amirkabir University of Technology, Tehran, Iran

2) Shahid Chamran College of Kerman/Technical & Vocational University, Tehran, Iran

3) Department of Management and Engineering, University of Padova, Vicenza, Italy

[Manuscript received 7 December 2012, in revised form 27 December 2013]

© The Chinese Society for Metals and Springer-Verlag Berlin Heidelberg

The effect of the distance between the notch tip and the position of the middle phase in the FGSs on the Charpy impact energy is investigated in the present paper. The results show that when the notch apex is close to the middle layer, the Charpy impact energy reaches its maximum value. This is due to the increment of the absorbed energy by plastic deformation ahead of the notch tip. On the other hand, when the notch tip is far from the middle layer, the Charpy impact energy strongly decreases. Another fundamental motivation of the present work is that for crack arrester configuration, no accurate mathematical or analytical modelling is available up to now. By considering the relationship between the Charpy impact energy and the plastic volume size, a new theoretical model has been developed to link the Charpy impact energy with the distance from the notch apex to the middle phase. This model is a simplified one and the effect of different shapes of the layers and the effect of microstructure on the mechanical properties and plastic region size will be considered in further investigation. The results of the new developed closed form expression show a sound agreement with some recent experimental results taken from the literature.

KEY WORDS: Charpy impact energy; Functionally graded steel; Crack divider configuration; Crack arrester configuration

1. Introduction

Functionally graded materials (FGMs) are multi-phase systems characterized by a chemical composition that, gradually varying in some directions, allow obtaining unique mechanical, thermal, electrical, optical, electromagnetic or biological properties, which distinguish them from the conventional composites^[1]. Functionally graded steels (FGSs) may be produced during welding of alloy steels. Jang *et al.*^[2] studied experimentally and numerically the effects of notch position on the failure behavior and energy absorption when the Charpy impact test was performed at -1 °C. Carbon steel plate with a thickness of 25 mm

was welded and specimens were fabricated from the welded plate. The Charpy impact tests were then performed on specimens having different notch positions varying within the heat affected zone (HAZ). A series of 3-D FE analyses which simulate the Charpy tests were also carried out in the same work.

Recently, FGSs have been produced by electrosag remelting (ESR)^[3]. Studies on transformation characteristics of FGSs produced from austenitic stainless steel and plain carbon steel have shown that as chromium, nickel, and carbon atoms diffuse at remelting stage, alternating regions with different transformation characteristics can be created in the material. By selecting appropriate arrangement and thickness of original electrodes made of ferritic and austenitic steels, composites with graded ferritic (α), and austenitic (γ) regions together with bainitic (β) or martensitic (M) layers can be obtained^[3].

In some previous studies, FGSs with different con-

† Corresponding author. Tel: +98 21 64543494; Fax: +98 21 66498441; E-mail address: hadi_salavati@aut.ac.ir (H. Samareh Salavati Pour)

figurations were produced and the tensile behavior was experimentally investigated and modeled. In particular, attention was paid to characterize the Vickers microhardness profile of the composite^[4]. Recently, Vickers microhardness profile of the $\alpha\beta\gamma$ FGSs was assessed with a model based on the strain gradient plasticity theory^[5]. The main advantage of these last models with respect to the previous ones^[4] is that the microhardness of each layer (*i.e.* microhardness profile) is not required for determining the mechanical properties of the FGSs.

Other important aspects are related to the characterization of FGSs under hot working conditions. A critical review of the constitutive descriptions for metals and alloys in hot working was extensively carried out in Ref. [7]. A large number of empirical and theoretical investigations have been carried out also on single phase steel^[8,9] and aluminium alloys^[10].

In Ref. [11], the flow stress of FGSs under hot compression loading was assessed by applying constitutive equations in combination with the rule of mixture. In this paper, a theoretical model has been proposed to assess the flow stress of the graded steels under hot compression conditions based on Reuss model for the overall strains^[11]. The brittle or quasi-brittle static failure of FGSs by considering a crack arrester configuration was studied by Barati *et al.*^[12]. The paper dealt with specimens made of $\gamma M \gamma$ FGS weakened by U-notches. As remarked in that work, when the notch tip is placed in the transition region between the austenite to martensite layers, the fracture load in FGS is greater than that of the homogeneous steel. On the other hand, when the notch is placed in the transition region between the martensite and austenite layers, the fracture load of the homogeneous steel is greater than that of FGS. In that study, Young's modulus and Poisson's ratio have been assumed to be constant, while the ultimate tensile strength and the fracture toughness K_{IC} vary exponentially through the specimen width. The effect of the notch depth on the critical fracture load has also been also investigated therein.

In Ref. [13, 14] the Charpy impact energy of crack divider specimens was measured experimentally. The obtained results showed that the Charpy impact energy of the specimens depended on the type and the volume fraction of the present phases. A theoretical model based on the rule of mixture, which correlates the Charpy impact energy of FGSs to the Charpy impact energy of the individual layers through Vickers microhardness of the layers, was obtained in Ref. [13]. Nazari and Aghazadeh Mohandesi^[14] obtained the Charpy impact energy for all layers of FGSs in the case of a crack divider configuration by using the relationship between the Charpy impact energy of each layer and the area under the stress-strain curve.

In addition, the Charpy impact energy^[15–17] as well as fracture toughness $J_{IC}^{[18]}$ of dual layer $\alpha\beta$ and $\beta\gamma$ FGSs in the form of crack divider configuration has

been assessed by using the strain gradient plasticity theory.

Following parallel tracks, the Charpy impact energy of FGSs produced by electroslag remelting in the form of crack arrester configuration has also been investigated in Ref. [1]. The results obtained in that study indicate that the notch tip position with respect to the bainite or martensite layer significantly affects the Charpy impact energy of FGSs. The closer the notch tip to the tougher layer (median bainitic or martensitic layers), the higher is the Charpy impact energy of the composite due to increment of energy absorbed by the plastic deformed zone ahead of the notch and vice versa. Empirical relationships have been determined to correlate the Charpy impact energy of FGSs to the morphology of each layer^[1]. As stated in Ref. [1] for arrester configuration, no precise mathematical modelling was presented except that done by using finite element simulation.

Following Ref. [15, 16], the Charpy impact energy of FGS composed of stacked ferritic or austenitic layers produced by electroslag remelting has been modeled by finite element method in both crack divider and crack arrester configurations^[19]. The yield stress of each layer has been related to the density of the dislocations of that layer and by assuming Holloman relation for the corresponding stress-strain curve, tensile strength and tensile strain of that layer were determined. Cubic elements were joined together to build the standard Charpy impact specimen. The stress-strain curve obtained from strain gradient plasticity theory was fed into the model as constitutive behavior. After applying the impact loading, a relatively good agreement between experimental results and those obtained from the simulation has been observed. As clearly stated in Ref. [19] for arrester configuration, no accurate mathematical modelling was presented except that done by using finite element simulation. This is an actual gap in the literature. The main aim of the present work is to fill this gap and provide a new analytical model for the assessment of the Charpy impact energy of FGSs in the form of crack arrester configuration.

In Ref. [20], the effect of the distance between the notch tip and the position of the median phase on the Charpy impact energy is investigated. The results show that when the notch apex is close to the median layer the impact energy reaches its maximum value due to the increment of the absorbed energy by plastic deformation ahead of the notch tip. On the other hand when the notch apex is far from the median layer, the impact energy strongly decreases. In the present work, the main objective is not to provide such empirical relations. Instead, the aim is to utilize these relations to develop a model to predict Charpy impact energy; because, there is not a direct relation to calculate the Charpy impact energy. The outputs of the proposed model for $\alpha\beta\gamma$ FGSs are compared with the experimental results taken from the recent

work^[1] and show a sound agreement.

2. Theoretical Background

Crack divider and crack arrester configurations are two common forms for FGS. Crack divider configuration is the case that the plane which contains the notch tip is perpendicular to the layers and crack arrester configuration is the case that the plane which contains the notch tip is parallel to the layers. These configurations for a three-layer $\alpha\beta\gamma$ FGSs respect to the Charpy impact test specimen are illustrated in Fig. 1. The Charpy impact energy of FGSs produced by electroslag remelting composed of graded ferrite and austenite layers together with bainite or martensite intermediate layer in the form of crack arrester configuration has been investigated in Ref. [1]. The experimental results obtained in that study indicated that the notch tip position with respect to bainite or martensite layer significantly affects the Charpy impact energy. The closer the notch tip to the tougher layer, the higher the Charpy impact energy of the

composite due to increment of energy absorbed by plastic deformation zone ahead of the notch and vice versa.

The Charpy impact energy of the studied composites depends on the notch tip position in alpha or gamma regions in addition to its distance from bainite or martensite layers. When the notch tip is far enough from the middle layer the Charpy impact energy is higher if the notch tip is in the austenitic region while it decreases if the notch apex is placed in the ferritic region of the specimen.

Empirical relationships have been determined to correlate the Charpy impact energy of functionally graded steels to the morphology of the layers. The relationship between Vickers micro hardness and the Charpy impact energy is as follows^[13]:

$$CV = A \frac{CV}{VH} + B \quad (1)$$

where CV denotes Charpy impact energy; VH stands for Vickers microhardness; and A and B are material constants.

Considering the linear relationship between the Vickers hardness and the yield strength^[4], Eq. (1) can be re-written as follows:

$$CV = A' \frac{CV}{\sigma_y} + B' \quad (2)$$

where σ_y is yield strength and A' and B' are constants. The right-hand side of Eq. (2) can be explicitly expressed as a function of the term $(K_{IC}/\sigma_y)^2$ according to following expression between the fracture toughness, yield strength and Charpy impact energy^[20,21]:

$$C \frac{CV}{\sigma_y} + D = \beta (K_{IC}/\sigma_y)^2 \quad (3)$$

where K_{IC} is the fracture toughness; β is a constant; C and D are materials constants. The term $(K_{IC}/\sigma_y)^2$ quantifies the size of the plastic region. In fact, the plastic radius, $r_y = \alpha (K_{IC}/\sigma_y)^2$, is proportional to $(K_{IC}/\sigma_y)^2$ (see Ref. [22]). Now, the combination of Eq. (2) and Eq. (3) allow us to suppose that the Charpy impact energy is proportional to r_y . Then, the simple relationship between the Charpy impact energy and the size of the plastic region can be written in the following form:

$$CV = N_1 r_y + N_2 \quad (4)$$

where N_1 and N_2 are constants which depend on the material. Eq. (4) is the basis of the proposed model because it shows the proportionality between the Charpy impact energy and the size of the plastic radius.

Considering the von Mises yield criterion, the following expression is valid for the plastic radius of a homogenous material under plane strain conditions and mode I loading^[22]:

Fig. 1 Schematic of the crack divider configuration (a), crack arrester configuration with the notch tip in ferritic graded region (b) and crack arrester configuration with the notch tip in austenitic graded region (c)

$$r_y = k \left(\frac{K_{IC}}{\sigma_y} \right)^2 \times \cos^2 \left(\frac{\theta}{2} \right) \times \left[(1 - 2\nu)^2 + 3 \sin^2 \left(\frac{\theta}{2} \right) \right] \quad (5)$$

where ν is Poisson's ratio and k is a constant.

3. New Proposed Model

Since, there is not a direct relation to calculate the Charpy impact energy; Eq. (4) has been utilized to obtain a new model to predict the Charpy impact energy of FGS in crack arrester configuration. Eq. (5) shows that the plastic region size varies as a function of θ .

In order to compensate the effect of the angle on the size of the plastic region in Eq. (5), the following new equation is proposed here to assess the Charpy impact energy:

$$CV_{FG} = N_2 + \left[\frac{A_{FG}}{A_H} \right]^{\frac{1}{2}} \times (CVH - N_2) \quad (6)$$

According to Eq. (6), the Charpy impact energy of a FGS, CV_{FG} , depends on the distance of the notch tip from the middle phase, d , and on the type of the considered composite. The term CVH is the Charpy impact energy of the layer containing the notch tip, A_H is the area of the plastic region for a homogeneous specimen composed exclusively of the material forming the layer containing the notch tip, A_{FG} is the area of the plastic region for a FGS specimen in which the mechanical properties vary along the direction of the notch bisector line and N_2 is a constant that was determined by imposing appropriate boundary conditions.

The mechanical properties as well as the Charpy impact energy of the ferritic and austenitic layers considered in the present paper have been determined experimentally in the literatures^[4,13,23] and were used as initial boundary conditions in the present work to characterize and assess the behaviour of the crack arrester specimen (see Table 1). Besides, the corresponding Charpy impact energy for entire layers in the case of a crack divider configuration has been determined numerically by Nazari and Najafi^[16].

As Fig. 1(a) shows, the divider configuration contains the graded layers which are placed along the X_1 direction from the outer edges (corresponding to single phase ferritic and austenitic steels) to the middle bainitic layer. While, in an arrester $\alpha\beta\gamma$ specimen, the notch tip can be located at either ferritic (Fig. 1(b)) or austenitic graded region (Fig. 1(c)). In this paper, parameter d presents the position of the notch tip respect to the middle β layer in arrester configuration. The ferritic and austenitic layers located in the edges of the divider specimen are ideally overlapped to the arrester specimen considered in the present contribution.

In order to determine the mechanical properties of the layer containing the notch tip, it has been supposed that the fracture toughness and yield strength vary exponentially with respect to the direction X_1 in the graded region:

$$K_{IC}(x) = D_1 \exp(D_2 x) \quad (7)$$

$$\sigma_y(x) = D_3 \exp(D_4 x) \quad (8)$$

where x is the distance of the layer from the subjective divider specimen edge, and D_i are constants that describe the variation of the mechanical properties inside the material and can be obtained by imposing the appropriate boundary conditions according to Table 1. Constants D_i are summarised in Table 2.

Note that the position of the layer close to the notch tip in the arrester configuration is indicated as $x^* - d$ in which x^* is the distance of the middle bainitic layer from the subjective divider specimen edge (corresponding to single phase ferritic and austenitic steels), which is ideally overlapped to the crack arrester specimen to impose the boundary conditions of the theoretical model proposed here.

Now, by integrating Eq. (5), the plastic area for the homogeneous material, A_H , can be derived:

$$\begin{aligned} A_H(d) &= \int_{\theta=0}^{\theta^*} r_H^2(\theta) d\theta = k^2 \left(\frac{K_{IC}(d)}{\sigma_y(d)} \right)^4 \int_{\theta=0}^{\theta^*} f^2(\theta) d\theta \\ &= k^2 \times \left(\frac{D_1}{D_3} \right)^4 \times \exp(4(D_2 - D_4)(x^* - d)) \times I_1 \quad (9) \end{aligned}$$

Table 1 Mechanical properties of single phase steels present in the considered FGSs

Single phase	Yield strength (MPa) ^[4]	Charpy impact energy CV (J) ^[13]	K_{IC} (MPa·m ^{0.5}) ^[23]	Poisson's ratio ν
Ferritic	245	64	45.72	0.33
Austenitic	200	140	107.77	0.33
Bainitic	1025	108	82.08	0.33
Martensitic	1440	11	6.09	0.33

Table 2 Material parameters in Eqs. (7) and (8)

Notch tip position	D_1 (MPa·m ^{0.5})	D_2 (m ⁻¹)	D_3 (MPa)	D_4 (m ⁻¹)
notch in the ferritic region of $\alpha\beta\gamma$	45.7	133.0	245.0	357.8
notch in the austenitic region of $\alpha\beta\gamma$	107.8	-61.9	200.0	408.5

In Eq. (9), I_1 is angular integral depending on Poisson ratio and can be expressed as follows:

$$I_1(\nu) = \int_{\theta=0}^{\pi} \left\{ \cos^2 \frac{\theta}{2} [(1-2\nu)^2 + 3 \sin^2 \frac{\theta}{2}] \right\}^2 d\theta \quad (10)$$

For $\nu=0.33$, I_1 is equal to 2.2594. This value is independent of the distance d and the type of material. The size of the plastic area for the homogeneous materials is plotted in Fig. 2 as a function of the distance d between the notch tip and the middle bainitic layer.

Fig. 2(a) refers to the case in which the notch tip is placed in a homogenous ferritic region while Fig. 2(b) corresponds to a notch tip located in a homogeneous austenitic region. With the aim to determine the plastic size for the FG material, due to the fact that $(K_{IC}/\sigma_y)^2$ varies gradually ahead of the notch tip, the influence of the angle θ and of the radial coordinate r have to be considered simultaneously.

By considering a specific layer at a distance x' from the notch in the crack arrester configuration, the layer is placed at a distance $x = x' + x^* - d$ from the ferritic edge of the ideally overlapped cracked divider specimen (see Fig. 3).

In Fig. 3, $r_{FG(i)}$ and $\theta(i)$ are the radius of the plastic area in the FG material and the angular coordinate respectively.

In the case of a FG material, Eq. (5) can be written as follows:

$$r_{FG(i)} = k \times H(x'_i) \times f(\theta(i)) \quad (11)$$

$$H(x'_i) = (K_{IC}(x'_i)/\sigma_y(x'_i))^2 \quad (12)$$

Fig. 2 Variation of the plastic area of the homogenous material (AH) versus the distance of the notch tip from middle layer when the notch tip is placed: (a) in the homogeneous ferritic region; (b) in the homogeneous austenitic region of $\alpha\beta\gamma$ FGS

Fig. 3 Profile of the plastic region ahead of the notch tip in the graded region of $\alpha\beta\gamma$ FGS in the form of crack arrester configuration

where $K_{IC}(x'(i))$ and $\sigma_y(x'(i))$ are the fracture toughness and the yield stress corresponding to the i^{th} layer and that vary in the material according to Eq. (7) and Eq. (8). The distance $x'(i)$ of the i^{th} layer from the notch tip can be expressed as follows:

$$x'_i = r_{FG(i)} \cos(\theta(i)) \quad (13)$$

By combining Eq. (11) and Eq. (13), one obtains:

$$\frac{x'_i}{H(x'_i)} = \alpha \cdot f(\theta(i)) \cos(\theta(i)) \quad (14)$$

$$H(x'_i) = \left(\frac{K_{IC}(x'_i)}{\sigma_y(x'_i)} \right)^2 = \left(\frac{D_1}{D_3} \right)^2 \exp(2(D_2 - D_4)(x^* - d + x'_i)) \quad (15)$$

By substituting the term $\exp(x) = 1 + x + O(x^2)$ in Eq. (15) according to Taylor's series expansion and by neglecting the higher order terms, $x'(i)$ can be expressed in the following form:

$$x'_i = \frac{\alpha \times \eta_1 \times f(\theta(i)) \cos(\theta(i))}{1 + \alpha \times \eta_2 \times f(\theta(i)) \cos(\theta(i))} \quad (16)$$

In Eq. (16), η_1 and η_2 are expressed as follows:

$$\eta_1 = \left(\frac{D_1}{D_3} \right)^2 [1 + 2(D_2 - D_4)(x^* - d)] \quad (17)$$

$$\eta_2 = -2(D_2 - D_4) \left(\frac{D_1}{D_3} \right)^2 \quad (18)$$

Substituting the expression for $x'(i)$ in Eq. (11), the radius $r_{FG(i)}$ can be finally derived:

$$r_{FG(i)} = \alpha \times \left(\frac{D_1}{D_3} \right)^2 \times [1 + 2(D_2 - D_4)(x^* - d + \frac{\alpha \times \eta_1 \times f(\theta(i)) \cos(\theta(i))}{1 + \alpha \times \eta_2 \times f(\theta(i)) \cos(\theta(i))})] \times f(\theta(i)) \quad (19)$$

By integrating the radius $r_{FG(i)}$ it is possible to determine the plastic area for the FG material, A_{FG} :

$$A_{FG}(d) = \int_{\theta=0}^{\theta=\pi} r_{FG}^2(\theta) d\theta = \alpha^2 \times \left(\frac{D_1}{D_3}\right)^4 \times \{ [1 + 4(D_2 - D_4)(x^* - d) + 4(D_2 - D_4)^2 \times (X^* - d)^2] \times I_1 + 4\alpha\eta_1 \times [2(D_2 - D_4)^2(x^* - d) + (D_2 - D_4)] \times I_3 + 4\alpha^2\eta_1^2 \cdot (D_2 - D_4)^2 \times I_2 \} \quad (20)$$

In Eq. (20), I_2 and I_3 correspond to the integrals reported below:

$$I_2 = \int_{\theta=0}^{\theta=\pi} \frac{f(\theta)^4 \cos^2(\theta)}{[1 + \alpha\eta_2 f(\theta) \cos(\theta)]^2} d\theta \quad (21)$$

$$I_3 = \int_{\theta=0}^{\theta=\pi} \frac{f^3(\theta) \cos(\theta)}{1 + \alpha\eta_2 f(\theta) \cos(\theta)} d\theta \quad (22)$$

Integrals I_2 and I_3 contain parameter η_2 which depends on the FGS considered.

In fact, η_2 is a function of the distance x^* from the austenitic or ferritic edge from the middle layer (bainitic) and then it is influenced by the boundary conditions imposed in the model. The boundary conditions, in fact, depend on the type of the considered FG material but do not depend on the distance d .

Integrals I_2 and I_3 have to be calculated for each graded region:

$$I_{2,3} = I(\text{composite type, boundary conditions}) \quad (23)$$

Parameter η_1 takes into account the effect of the notch tip position (distance d) inside the considered FG material

$$\eta_1 = \eta_1(\text{composite type, boundary conditions, } d) \quad (24)$$

The variation of the plastic area of FG materials as a function of the distance d between the notch tip from the middle bainitic layer is shown in Fig. 4.

Fig. 4(a) shows the case in which the notch tip is placed in the ferritic region while Fig. 4(b) refers to the case of a notch located in the austenitic region. The constant N_2 (see Eq. (6)) can be obtained as follows:

$$\frac{CVH_1 - N_2}{CVH_2 - N_2} = \frac{\sqrt{A_{H(1)}}}{\sqrt{A_{H(2)}}} \quad (25a)$$

$$N_2 = \frac{\sqrt{A_{H(1)}}CVH(2) - \sqrt{A_{H(2)}}CVH(1)}{\sqrt{A_{H(1)}} - \sqrt{A_{H(2)}}} = \frac{(K_{IC}/\sigma_y)_{(1)}^2 CVH(2) - (K_{IC}/\sigma_y)_{(2)}^2 CVH(1)}{(K_{IC}/\sigma_y)_{(1)}^2 - (K_{IC}/\sigma_y)_{(2)}^2} \quad (25b)$$

In Eq. (25), the subscripts (1) and (2) correspond to the boundary layers of the considered region which includes the notch tip. In the case of a $\alpha\beta\gamma$ FG material and a notch tip placed in the ferritic region subscript (1) refers to the mechanical properties of the ferritic

Fig. 4 Variation of the plastic area of FGS (AFG) versus the notch tip distance from the middle phase for: (a) the ferritic region in $\alpha\beta\gamma$ composite and; (b) the austenitic region of $\alpha\beta\gamma$ composite

edge while subscript (2) refers to the bainitic mid layer.

By substituting Eqs. (9), (20) and (25b) in Eq. (6), the final expression of the Charpy impact energy for the FGS as a function of the distance between the notch tip and the middle phase has been obtained according to the following expression:

$$CV_{FG} = \frac{(K_{IC}/\sigma_y)_{(1)}^2 CVH(2) - (K_{IC}/\sigma_y)_{(2)}^2 CVH(1)}{(K_{IC}/\sigma_y)_{(1)}^2 - (K_{IC}/\sigma_y)_{(2)}^2} + \left\{ \frac{\alpha^2}{\exp[4 \times (\ln[\frac{K_{IC(2)}}{K_{IC(1)}}] - \ln[\frac{\sigma_{y(2)}}{\sigma_{y(1)}}]) \times \frac{x^* - d}{x^*}] \times I_1} \times \langle [1 + 4 \times (\ln[\frac{K_{IC(2)}}{K_{IC(1)}}] - \ln[\frac{\sigma_{y(2)}}{\sigma_{y(1)}}]) \times \frac{x^* - d}{x^*}] + 4 \times (\ln[\frac{K_{IC(2)}}{K_{IC(1)}}] - \ln[\frac{\sigma_{y(2)}}{\sigma_{y(1)}}])^2 \times (\frac{x^* - d}{x^*})^2 \rangle \times I_1 + 4\alpha\eta_1 \times [2 \times (\ln[\frac{K_{IC(2)}}{K_{IC(1)}}] - \ln[\frac{\sigma_{y(2)}}{\sigma_{y(1)}}])^2 \times \frac{x^* - d}{x^*} + \frac{1}{x^*} \times (\ln[\frac{K_{IC(2)}}{K_{IC(1)}}] - \ln[\frac{\sigma_{y(2)}}{\sigma_{y(1)}}]) \times I_3 + 4(\frac{\alpha\eta_1}{x^*})^2 \times (\ln[\frac{K_{IC(2)}}{K_{IC(1)}}] - \ln[\frac{\sigma_{y(2)}}{\sigma_{y(1)}}])^2 \times I_3 \rangle^{\frac{1}{2}} \times [CVH - \frac{(K_{IC}/\sigma_y)_{(1)}^2 CVH(2) - (K_{IC}/\sigma_y)_{(2)}^2 CVH(1)}{(K_{IC}/\sigma_y)_{(1)}^2 - (K_{IC}/\sigma_y)_{(2)}^2}] \right\} \quad (26)$$

Eq. (26) gives the Charpy impact energy of the graded materials and can be applied to different types of FGSs ($\gamma M\gamma$, $\alpha\beta\gamma M\gamma$, *etc.*). In this paper, just ferrite-bainite-austenite ($\alpha\beta\gamma$) FGS was considered. The term CV_{FG} depends on the boundary conditions applied on the layers 1 and 2 which include the notch tip and also the distance d between the notch tip and the middle layer.

$$CV_{FG} = CV(\text{composite, boundary, condition, position}) \quad (27)$$

By considering the values summarised in Table 1, the constants required for the application of Eq. (26) have been determined. These are for ferrite -bainite and notch in the ferritic region (subscript (1) refers to the ferritic edge while subscript (2) refers to the bainitic mid layer):

$K_{IC(1)}=45.72 \text{ MPa}\cdot\text{m}^{0.5}$, $K_{IC(2)}=82.08 \text{ MPa}\cdot\text{m}^{0.5}$, $\sigma_{y(1)} = 245 \text{ MPa}$, $\sigma_{y(2)} = 1025 \text{ MPa}$, $CVH_{(1)} = 64 \text{ J}$, $CVH_{(2)}=108 \text{ J}$, $x^*=0.0044 \text{ m}$.

These are for austenite-bainite and notch in the austenitic region (subscript (1) refers to the austenitic edge while subscript (2) refers to the bainitic mid layer):

$K_{IC(1)} = 107.77 \text{ MPa}\cdot\text{m}^{0.5}$, $K_{IC(2)} = 82.08 \text{ MPa}\cdot\text{m}^{0.5}$, $\sigma_{y(1)} = 200 \text{ MPa}$, $\sigma_{y(2)} = 1025 \text{ MPa}$, $CVH_{(1)} = 140 \text{ J}$, $CVH_{(2)} = 108 \text{ J}$, $x^*=0.004 \text{ m}$.

The constants D_1 , D_2 , D_3 and D_4 are evaluated as follows:

$$\begin{aligned} D_1 &= K_{IC(1)} \\ D_2 &= \frac{1}{x^*} \ln\left(\frac{K_{IC(2)}}{K_{IC(1)}}\right) \\ D_3 &= \sigma_{Y(1)} \\ D_4 &= \frac{1}{x^*} \ln\left(\frac{\sigma_{y(2)}}{\sigma_{y(1)}}\right) \end{aligned} \quad (28)$$

The main parameters for the application of Eq. (26) are summarised in Table 3. Under plane strain conditions, the value of α according to the von-Mises criterion is $1/(6\pi)$ (see Eq. (22)).

In Eq. (26), CVH is the Charpy impact energy of the homogeneous layer placed at the distance $(x^* - d)$ from the crack divider specimen edge.

The Charpy impact energy of the homogeneous case, CVH , has been evaluated in^[16]. By substituting CVH in Eq. (26), the Charpy impact energy of the FGS, CV_{FG} , has been obtained. The variation of the Charpy impact energy both for the homogeneous material and for the FGS as a function of the distance from the notch tip to the median layer are shown and compared in Fig. 5.

Fig. 5 shows that the Charpy impact energy when the notch is in the ferritic region of the composite is always greater than the value obtained considering the

homogenous material characterized by the mechanical properties of the layer containing the notch tip.

On the other hand, when the notch is in the austenitic region, the Charpy impact energy of the FGS is always lower than that corresponding to the homogenous specimen. In order to investigate the accuracy of the model, the results are compared with the experimental data taken from^[1]. The values of the Charpy impact energy for three values of the distance d are reported in Table 4 and compared with the proposed model.

The comparison shows that Eq. (26) allows assessing the Charpy impact energy with good accuracy in the case of the considered $\alpha\beta\gamma$ composite. In general viewpoint, application of the approach is sketch in a

Fig. 5 Variation of the impact energy as a function of distance d ($\alpha\beta\gamma$ steel): (a) notch tip in the ferritic region: FGS versus homogeneous ferritic material; (b) notch tip in the austenitic region: FGS versus homogeneous austenitic material

Table 3 Parameters for application of Eq. (26) in the considered cases

Notch in the ferritic region of $\alpha\beta\gamma$						
d (mm)	I_1	I_2	I_3	η_1 (m)	η_2	CVH (J)
1	2.2594	0.0104	0.1513	-0.01133	12.67	99.98
2	2.2594	0.0104	0.1513	0.00134	12.67	86.43
3	2.2594	0.0104	0.1513	0.01401	12.67	73.40
Notch in the ferritic region of $\alpha\beta\gamma$						
d (mm)	I_1	I_2	I_3	η_1 (m)	η_2	CVH (J)
1	2.2594	2.877e-05	0.0081	-0.54022	276.78	113.51
2	2.2594	2.877e-05	0.0081	-0.26344	276.78	123.23
3	2.2594	2.877e-05	0.0081	0.01334	276.78	131.01

flowchart as shown in Fig. 6.

Note that in the present work, the plastic zone is assumed in the vicinity of the crack and only the position of the notch is considered variable in the development of Eq. (26).

The variability of the Charpy impact energy as a function of the notch geometry (notch depth and angle) will be considered in a next contribution. FEM analysis could also be done to evaluate the accuracy of the results.

Table 4 Comparison between the Charpy impact energy CVFG of $\alpha\beta\gamma$ FGS assessed by the present model and experimental data for different distances of the notch tip from the median phase

d (mm)	FGS type			
	Notch in the ferritic region		Notch in the austenitic region	
	Experimental ^[1]	CV _{FG} Eq. (26)	Experimental ^[1]	CV _{FG} Eq. (26)
1	99	102.68	111	112.05
2	85	84.42	119	121.75
3	75	79.22	125	128.37
Mean error		4.45		1.98
Standard variance		0.84		0.75

Fig. 6 Flow chart for the application of the proposed method

4. Conclusions

The main findings of the present work can be summarised as follows:

A new analytical predictive model is proposed to evaluate the Charpy impact energy of FGSs in crack arrester configuration. Different distances of the notch tip from the median phase of the composite are considered as well as different sizes of the plastic region ahead of the notch tip.

The new model has been applied to $\alpha\beta\gamma$ FGS and the results have been compared with those reported in the literature for the same material.

The average deviation between the theoretical values and the experimental data taken from the literature is 4.45% and 1.98% when the notch tip is located in the ferritic and austenitic region, respectively.

The Charpy impact energy of the FGSs is compared with the specimen made of the homogeneous material corresponding to the layer that includes the notch tip. The results show that the Charpy impact energy when the notch is in the ferritic region of $\alpha\beta\gamma$ FGS is higher than that corresponding to the homogeneous ferritic material. On the other hand, when the notch is located in the austenitic region, the Charpy impact energy of graded materials is always lower than that corresponding to the homogenous austenitic material.

With the aim to improve the present model, further developments will consider the influence of the strain rate and the effect of the notch geometry (notch depth and notch angle) on the Charpy impact energy of FGSs.

REFERENCES

- [1] A. Nazari and J. Aghazadeh, *J. Mater. Eng. Perform.* **19** (2012) 1058.
- [2] Y.C. Jang, J.K. Hong, J.H. Park, D.W. Kim and Y. Lee, *J. Mater. Proc. Technol.* **201** (2008) 419.
- [3] J. Aghazadeh Mohandesi and M.H. Shahossinie, *Metall. Trans. A* **36** (2005) 3471.
- [4] J. Aghazadeh Mohandesi, M.H. Shahossinie and R. Parastar Namin, *Metall. Mater. Trans. A* **37** (2006) 2125.
- [5] A. Nazari, J. Aghazadeh Mohandesi and S. Tavareh, *Comput. Mater. Sci.* **50** (2011) 1781.
- [6] A. Nazari, J. Aghazadeh Mohandesi and S. Tavareh, *Comput. Mater. Sci.* **50** (2011) 1791.
- [7] Y.C. Lin and X.M. Chen, *Mater. Des.* **32** (2011) 1733.
- [8] Y.C. Lin, M.S. Chen and J. Zhong, *Comput. Mater. Sci.* **42** (2008) 470.
- [9] Y.C. Lin and G. Liu, *Comput. Mater. Sci.* **48** (2010) 54.
- [10] Y.C. Lin, Y.C. Xia, X.M. Chen and M.S. Chen, *Comput. Mater. Sci.* **50** (2010) 227.
- [11] M. Abolghasemzadeh, H. Samareh Salavati Pour, F. Berto and Y. Alizadeh, *Mater. Sci. Eng. A* **534** (2012) 329.
- [12] E. Barati, Y. Alizadeh and J. Aghazadeh Mohandesi, *Eng. Fract. Mech.* **77** (2010) 3341.
- [13] A. Nazari and J. Aghazadeh Mohandesi, *J. Mater. Sci. Technol.* **25** (2009) 847.
- [14] A. Nazari and J. Aghazadeh Mohandesi, *J. Mater. Sci. Technol.* **26** (2010) 1377.
- [15] A. Nazari and S.M.M. Najafi, *Comput. Mater. Sci.* **50** (2011) 3178.
- [16] A. Nazari and S.M.M. Najafi, *Comput. Mater. Sci.* **50** (2011) 3218.
- [17] A. Nazari, *Comput. Mater. Sci.* **51** (2012) 281.
- [18] A. Nazari, *Comput. Mater. Sci.* **50** (2011) 3238.
- [19] A. Nazari, *Comput. Mater. Sci.* **51** (2012) 13.
- [20] H. Samareh Salavati Pour, Y. Alizadeh, F. Berto and M. Abolghasemzadeh, *Proc 4th International Conference on Crack Path*, Gaeta, Italy, 2012, pp.505-512.
- [21] Y.J. Chao, J.D. Ward and R.G. Sands, *Mater. Des.* **28** (2007) 551.
- [22] R.W. Hertzberg, *Deformation and Fracture Mechanics of Engineering Materials*, 4th Ed., New York, John Wiley, 1996, p.1.
- [23] A. Nazari, J. Aghazadeh Mohandesi and S. Riahi, *J. Mater. Eng. Perform.* **21** (2012) 558.